S-wave bottom tetraquarks

S.M. Gerasyuta [1,2] and V.I. Kochkin [1]

[1] Department of Theoretical Physics, St. Petersburg State University, 198904, St. Petersburg, Russia

[2] Department of Physics, LTA, 194021, St. Petersburg, Russia

E-mail: gerasyuta@SG6488.spb.edu

Abstract

The relativistic four-quark equations are found in the framework of coupled-channel formalism. The dynamical mixing of the meson-meson states with the four-quark states is considered. The four-quark amplitudes of the tetraquarks, including u, d, s and bottom quarks, are constructed. The poles of these amplitudes determine the masses and widths of S-wave bottom tetraquarks.

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I. Introduction.

The remarkable progress at the experimental side has opened up new challenges in the theoretical understanding of heavy flavor hadrons. The observation of $X(3872)$ resonance [1] has been confirmed by CDF [2], D0 [3] and BaBar Collaboration [4]. Belle Collaboration observed the $X(3940)$ in double-charmonium production in the reaction $e^+e^- \to J/\psi + X$ [5]. The state, designated as $X(4160)$, was reported by the Belle Collaboration in Ref. 6. The fact that the newly found states do not fit quark model calculations [7] has opened the discussion about the structure of such states. Maiani et al. advocate a tetraquark explanation for the $X(3872)$ [8, 9]. On the other hand, the mass of $X(3872)$ is very close to the threshold of D^*D and, therefore, it can be interpreted as molecular state [10 – 15]. In our paper [16] the dynamical mixing between the meson-meson states and the four-quark states is considered. Taking the $X(3872)$ and $X(3940)$ as input [17, 18] we predicted the masses and widths of S-wave tetraquarks with open and hidden charm.

In the recent papers [19 – 21], the relativistic three-quark equations for the excited baryons are found in the framework of the dispersion relations technique. We have used the orbital-spin-flavor wave functions for the construction of integral equations. We calculated the mass spectra of P-wave single, double, and triple charmed baryons using the input four-fermion interaction with quantum numbers of the gluon [21].

In the present paper the relativistic four-quark equations for the tetraquarks with hidden and open bottom in the framework of the dispersion relation technique are found. We searched for the approximate solution of these equations by taking into account two-particle, triangle and four-particle singularities, all the weaker ones being neglected. The masses and widths of the low-lying bottom tetraquarks are calculated.

After this introduction, we obtain the relativistic four-particle equations which describe the interaction of the quarks (Sec. II). Section III is devoted to a calculation of the masses and widths of S-wave bottom tetraquarks (Tables I, II and III).

II. Four-quark amplitudes for the S-wave bottom tetraquarks.

We derive the relativistic four-quark equations in the framework of the dispersion relations technique. The correct equations for the amplitude are obtained by taking into account subsystems with the smaller number of particles. Then one should represent a four-particle amplitude as a sum of six subamplitudes:

$$A = A_{12} + A_{13} + A_{14} + A_{23} + A_{24} + A_{34} \,. \tag{1}$$

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams.

We need to consider only one group of diagrams and the amplitude corresponding to them, for example A_{12}. We shall consider the derivation of the relativistic generalization of the Faddeev-Yakubovsky approach [22, 23] for the tetraquarks.

The four-quark amplitude of $b\bar{b}u\bar{u}$ tetraquark includes the quark amplitudes with quantum numbers of 0^{-+} and 1^{--} mesons. The set of diagrams associated with the amplitude A_{12} can further be broken down into five groups corresponding to subamplitudes: $A_1(s, s_{12}, s_{34})$, $A_2(s, s_{23}, s_{14})$, $A_3(s, s_{23}, s_{123})$, $A_4(s, s_{34}, s_{234})$, $A_5(s, s_{12}, s_{123})$, if we consider the tetraquark with the $J^{pc} = 2^{++}$.

Here s_{ik} is the two-particle subenergy squared, s_{ijk} corresponds to the energy squared of particles i, j, k and s is the system total energy squared.

In order to represent the subamplitudes $A_1(s, s_{12}, s_{34})$, $A_2(s, s_{23}, s_{14})$, $A_3(s, s_{23}, s_{123})$, $A_4(s, s_{34}, s_{234})$ and $A_5(s, s_{12}, s_{123})$ in the form of dispersion relations it is necessary to define the amplitudes of quark-antiquark interaction $a_n(s_{ik})$. The pair quarks amplitudes $q\bar{q} \to q\bar{q}$ are calculated in the framework of the dispersion N/D method with the input four-fermion interaction [24 – 26] with quantum numbers of the gluon [27]. The regularization of the dispersion integral for the D-function is carried out with the cutoff parameter Λ. The four-quark interaction is considered as an input [27]:

$$\begin{aligned} & g_V \left(\bar{q}\lambda I_f \gamma_\mu q\right)^2 + g_V^{(s)} \left(\bar{q}\lambda I_f \gamma_\mu q\right)\left(\bar{s}\lambda \gamma_\mu s\right) + g_V^{(ss)} \left(\bar{s}\lambda \gamma_\mu s\right)^2 \\ & + g_V^{(b)} \left(\bar{q}\lambda I_f \gamma_\mu q\right)\left(\bar{b}\lambda \gamma_\mu b\right) + g_V^{(bb)} \left(\bar{b}\lambda \gamma_\mu b\right)^2 + g_V^{(bs)} \left(\bar{b}\lambda \gamma_\mu b\right)\left(\bar{s}\lambda \gamma_\mu s\right) \,. \end{aligned} \tag{2}$$

Here I_f is the unity matrix in the flavor space (u, d). λ are the color Gell-Mann matrices. Dimensional constants of the four-fermion interaction g_V, $g_V^{(s)}$, $g_V^{(ss)}$, $g_V^{(b)}$, $g_V^{(bb)}$ and $g_V^{(bs)}$ are parameters of the model. In order to avoid an additional violation parameters, we introduce the scale shift of the dimensional parameters:

$$g = \frac{m^2}{\pi^2} g_V = \frac{(m+m_s)^2}{4\pi^2} g_V^{(s)} = \frac{m_s^2}{\pi^2} g_V^{(ss)} = \frac{(m_b+m)^2}{4\pi^2} g_V^{(b)} = \frac{m_b^2}{\pi^2} g_V^{(bb)} = \frac{(m_b+m_s)^2}{4\pi^2} g_V^{(bs)} \,. \tag{3}$$

$$\Lambda = \frac{4\Lambda(ik)}{(m_i + m_k)^2}. \tag{4}$$

m_i and m_k are the quark masses in the intermediate state of the quark loop $(i, k = q, s, b)$. Dimensionless parameters g and Λ are supposed to be constants which are

independent of the quark interaction type. The applicability of Eq. (2) is verified by the success of De Rujula-Georgi-Glashow quark model [28], where only the short-range part of Breit potential connected with the gluon exchange is responsible for the mass splitting in hadron multiplets.

We use the results of our relativistic quark model [27] and write down the pair quarks amplitude in the form:

$$a_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})}\,, \tag{5}$$

$$B_n(s_{ik}) = \int\limits_{(m_i+m_k)^2}^{\frac{(m_i+m_k)^2\Lambda}{4}} \frac{ds'_{ik}}{\pi}\frac{\rho_n(s'_{ik})G_n^2(s'_{ik})}{s'_{ik} - s_{ik}}\,. \tag{6}$$

Here $G_n(s_{ik})$ are the quark-antiquark vertex functions. The vertex functions are determined by the contribution of the crossing channels. The vertex functions satisfy the Fierz relations. All of these vertex functions are generated from g_V, $g_V^{(s)}$, $g_V^{(ss)}$, $g_V^{(b)}$, $g_V^{(bb)}$ and $g_V^{(bs)}$. $B_n(s_{ik})$, $\rho_n(s_{ik})$ are the Chew-Mandelstam functions with cutoff Λ and the phase spaces, respectively.

In the case in question, the interacting quarks do not produce a bound state; therefore, the integration in Eqs. (7) – (11) is carried out from the threshold $(m_i + m_k)^2$ to the cutoff $\Lambda(ik)$. The integral equation systems (the meson state $J^{pc} = 2^{++}$ for the $b\bar{b}u\bar{u}$) can be described as:

$$A_1(s, s_{12}, s_{34}) = \frac{\lambda_1 B_1(s_{12})B_1(s_{34})}{[1 - B_1(s_{12})][1 - B_1(s_{34})]} + 4\hat{J}_2(s_{12}, s_{34}, 1, 1)A_3(s, s'_{23}, s'_{123})\,, \tag{7}$$

$$\begin{aligned} A_2(s, s_{23}, s_{14}) &= \frac{\lambda_2 B_1(s_{23})B_1(s_{14})}{[1 - B_1(s_{23})][1 - B_1(s_{14})]} + 2\hat{J}_2(s_{23}, s_{14}, 1, 1)A_4(s, s'_{34}, s'_{234}) \\ &+ 2\hat{J}_2(s_{23}, s_{14}, 1, 1)A_5(s, s'_{12}, s'_{123})\,, \end{aligned} \tag{8}$$

$$\begin{aligned} A_3(s, s_{23}, s_{123}) &= \frac{\lambda_3 B_2(s_{23})}{[1 - B_2(s_{23})]} + 2\hat{J}_3(s_{23}, 2)A_1(s, s'_{12}, s'_{34}) \\ &+ \hat{J}_1(s_{23}, 2)A_4(s, s'_{34}, s'_{234}) + \hat{J}_1(s_{23}, 2)A_5(s, s'_{12}, s'_{123})\,, \end{aligned} \tag{9}$$

$$\begin{aligned} A_4(s, s_{34}, s_{234}) &= \frac{\lambda_4 B_2(s_{34})}{[1 - B_2(s_{34})]} + 2\hat{J}_3(s_{34}, 2)A_2(s, s'_{23}, s'_{14}) \\ &+ 2\hat{J}_1(s_{34}, 2)A_3(s, s'_{23}, s'_{234})\,, \end{aligned} \tag{10}$$

$$\begin{aligned} A_5(s, s_{12}, s_{123}) &= \frac{\lambda_5 B_2(s_{12})}{[1 - B_2(s_{12})]} + 2\hat{J}_3(s_{12}, 2)A_2(s, s'_{23}, s'_{14}) \\ &+ 2\hat{J}_1(s_{12}, 2)A_3(s, s'_{23}, s'_{123})\,, \end{aligned} \tag{11}$$

where λ_i, $i = 1, 2, 3, 4, 5$ are the current constants. They do not affect the mass spectrum of tetraquarks. $n = 1$ corresponds to a $q\bar{q}$-pair with $J^{pc} = 1^{--}$ in the 1_c color

state, and $n = 2$ defines the $q\bar{q}$-pairs corresponding to the S-wave tetraquarks with quantum numbers: $J^{pc} = 0^{++}$, 1^{++}, 2^{++}. We introduce the integral operators:

$$\hat{J}_1(s_{12}, l) = \frac{G_l(s_{12})}{[1 - B_l(s_{12})]} \int\limits_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda}{4}} \frac{ds'_{12}}{\pi} \frac{G_l(s'_{12})\rho_l(s'_{12})}{s'_{12} - s_{12}} \int\limits_{-1}^{+1} \frac{dz_1}{2}\,, \tag{12}$$

$$\hat{J}_2(s_{12}, s_{34}, l, p) = \frac{G_l(s_{12})G_p(s_{34})}{[1 - B_l(s_{12})][1 - B_p(s_{34})]} \int\limits_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda}{4}} \frac{ds'_{12}}{\pi} \frac{G_l(s'_{12})\rho_l(s'_{12})}{s'_{12} - s_{12}}$$
$$\times \int\limits_{(m_3+m_4)^2}^{\frac{(m_3+m_4)^2\Lambda}{4}} \frac{ds'_{34}}{\pi} \frac{G_p(s'_{34})\rho_p(s'_{34})}{s'_{34} - s_{34}} \int\limits_{-1}^{+1} \frac{dz_3}{2} \int\limits_{-1}^{+1} \frac{dz_4}{2}\,, \tag{13}$$

$$\hat{J}_3(s_{12}, l) = \frac{G_l(s_{12}, \tilde{\Lambda})}{[1 - B_l(s_{12}, \tilde{\Lambda})]} \frac{1}{4\pi} \int\limits_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\tilde{\Lambda}}{4}} \frac{ds'_{12}}{\pi} \frac{G_l(s'_{12}, \tilde{\Lambda})\rho_l(s'_{12})}{s'_{12} - s_{12}}$$
$$\times \int\limits_{-1}^{+1} \frac{dz_1}{2} \int\limits_{-1}^{+1} dz \int\limits_{z_2^-}^{z_2^+} dz_2 \frac{1}{\sqrt{1 - z^2 - z_1^2 - z_2^2 + 2zz_1z_2}}\,, \tag{14}$$

here l, p are equal to 1 or 2.

In Eqs. (12) and (14) z_1 is the cosine of the angle between the relative momentum of particles 1 and 2 in the intermediate state and the momentum of the particle 3 in the final state, taken in the c.m. of particles 1 and 2. In Eq. (14) z is the cosine of the angle between the momenta of particles 3 and 4 in the final state, taken in the c.m. of particles 1 and 2. z_2 is the cosine of the angle between the relative momentum of particles 1 and 2 in the intermediate state and the momentum of the particle 4 in the final state, is taken in the c.m. of particles 1 and 2. In Eq. (13) z_3 is the cosine of the angle between relative momentum of particles 1 and 2 in the intermediate state and the relative momentum of particles 3 and 4 in the intermediate state, taken in the c.m. of particles 1 and 2. z_4 is the cosine of the angle between the relative momentum of particles 3 and 4 in the intermediate state and that of the momentum of the particle 1 in the intermediate state, taken in the c.m. of particles 3 and 4.

In our model the integral equation system for the scalar open bottom ($J^{pc} = 0^{++}$ $\bar{b}u\bar{u}u$) can be described as:

$$A_1(s, s_{12}, s_{34}) = \frac{\lambda_1 B_2(s_{12})B_2(s_{34})}{[1 - B_2(s_{12})][1 - B_2(s_{34})]} + 2\hat{J}_2(s_{12}, s_{34}, 2, 2)A_3(s, s'_{23}, s'_{123})$$
$$+ 2\hat{J}_2(s_{12}, s_{34}, 2, 2)A_4(s, s'_{14}, s'_{124})\,, \tag{15}$$

$$A_2(s, s_{23}, s_{14}) = \frac{\lambda_2 B_1(s_{23})B_1(s_{14})}{[1 - B_1(s_{23})][1 - B_1(s_{14})]} + 2\hat{J}_2(s_{23}, s_{14}, 1, 1)A_3(s, s'_{34}, s'_{234})$$
$$+ 2\hat{J}_2(s_{23}, s_{14}, 1, 1)A_4(s, s'_{12}, s'_{123})\,, \tag{16}$$

$$
\begin{aligned}
A_3(s,s_{23},s_{123}) &= \frac{\lambda_3 B_3(s_{23})}{[1-B_3(s_{23})]} + 2\hat{J}_3(s_{23},3)A_1(s,s'_{12},s'_{34}) + \hat{J}_3(s_{23},3)A_2(s,s'_{12},s'_{34}) \\
&+ \hat{J}_1(s_{23},3)A_4(s,s'_{34},s'_{234}) + \hat{J}_1(s_{23},3)A_3(s,s'_{12},s'_{123})\,,
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
A_4(s,s_{14},s_{124}) &= \frac{\lambda_4 B_3(s_{14})}{[1-B_3(s_{14})]} + 2\hat{J}_3(s_{14},3)A_1(s,s'_{13},s'_{24}) + 2\hat{J}_3(s_{14},3)A_2(s,s'_{13},s'_{24}) \\
&+ 2\hat{J}_1(s_{14},3)A_3(s,s'_{14},s'_{134}) + 2\hat{J}_1(s_{14},3)A_4(s,s'_{14},s'_{134})\,,
\end{aligned} \tag{18}
$$

where λ_i, $i = 1,2,3,4$ are the current constants. We used the integral operators Eqs. (12) – (14). In the case in question, $n = 1$ determines a $q\bar{q}$-pair with $J^{pc} = 0^{-+}$ in the 1_c color state, $n = 2$ corresponds to $q\bar{q}$-pair with $J^{pc} = 1^{--}$ in the 1_c color state, and $n = 3$ defines the $q\bar{q}$-pair corresponding to the bottom tetraquarks with quantum numbers $J^{pc} = 0^{++}$.

We can pass from the integration over the cosines of the angles (Eqs. (12) – (14)) to the integration over the subenergies [29 – 31].

The solutions of the system of equations are considered as:

$$
\alpha_i(s) = F_i(s,\lambda_i)/D(s)\,, \tag{19}
$$

where zeros of $D(s)$ determinants define the masses of bound states of tetraquarks. $F_i(s,\lambda_i)$ determine the contributions of subamplitudes to the tetraquark amplitude.

III. Calculation results.

The model in consideration has only two parameters: the cutoff $\Lambda = 7.63$ and the gluon coupling constant $g = 1.53$. The experimental mass values of bottom tetraquarks are absent. Therefore these parameters are determined by fixing the bottom tetraquark masses for the $J^{pc} = 1^{++}$ $X_b(10300)$ and $J^{pc} = 2^{++}$ $X_b(10340)$ in the paper [32]. The widths of the bottom tetraquark are fitted by the fixing width $\Gamma_{2^{++}} = (39 \pm 26)\,MeV$ [33] for the S-wave tetraquark with the hidden charm $X(3940)$. The quark masses of model $m_{u,d} = 385\,MeV$ and $m_s = 510\,MeV$ coincide with the ordinary meson ones in our model [27]. We fix the mass $m_b = 4787\,MeV$. It is typical value for our calculation of bottom tetraquark masses ($m_b \geq \frac{1}{2}M(10340) - m_q$).

The masses of tetraquarks with hidden bottom are considered in Table I. The contributions of the subamplitudes also in Table I are given. The contributions of the four-quark states $Q\bar{q}\bar{Q}q$ are about 20% – 50% for the bottom tetraquarks. The functions $F_i(s,\lambda_i)$ (Eq. (19)) allow us to obtain the overlap factors f. We calculated the widths of the bottom tetraquarks using the formula $\Gamma \sim f^2 \times \rho$ [34], where ρ are the phase spaces for the reactions $X_b \to M_1 M_2$ (Table II).

The masses and widths of open bottom tetraquarks with the spin-parity $J^{pc} = 0^{++}$ in Table III are shown.

The results of calculations allow us to consider the tetraquark with hidden and open bottom as the narrow resonances. The calculated width of $X_b(10300)$ tetraquark with the mass $M = 10303\,MeV$ and the spin-parity $J^{pc} = 1^{++}$ is about $\Gamma_{1^{++}} = 43\,MeV$. The width of $X_b(9940)$ tetraquark with the mass $M = 9936\,MeV$ and the spin-parity $J^{pc} = 0^{++}$ is equal to $\Gamma_{0^{++}} = 96\,MeV$. We calculated also the width of $X_b(10340)$ tetraquark $\Gamma_{2^{++}} = 80\,MeV$ and the width of $X_b(10550)$ tetraquark ($b\bar{b}s\bar{s}$) $\Gamma_{2_s^{++}} =$

$58\,MeV$. The tetraquarks with the spin-parity $J^{pc} = 0^{++}$, 1^{++} $(s\bar{s}b\bar{b})$ have only the weak decays.

In our paper we predicted the tetraquark $(\bar{b}u\bar{u}u)$ with the mass $M = 5914\,MeV$ and width $\Gamma_{0^{++}} = 104\,MeV$. We calculated the mass of $X_b(6020)$ $M = 6017\,MeV$ and width $\Gamma_{0^{++}} = 69\,MeV$ (channels $B_s^0\eta$ and B^+K^-). The tetraquark $(\bar{b}su\bar{s})$ has the mass $M = 6122\,MeV$ and width $\Gamma_{0^{++}} = 48\,MeV$.

The tetraquarks with open bottom and the spin-parity $J^{pc} = 1^{++}$, 2^{++} have only the weak decays. In the open bottom sector (Table III) the scalar tetraquarks have relatively small widths $\sim 50 - 100\,MeV$, so in principle, these exotic states could be observed.

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Table I. Low-lying meson-meson state masses (MeV) of tetraquarks with hidden bottom and the contributions of subamplitudes to the tetraquark amplitudes (in percent).

Parameters of model: quark masses $m_u = 385\,MeV$, $m_s = 510\,MeV$, $m_b = 4787\,MeV$, cutoff parameter $\Lambda = 7.63$, gluon coupling constant $g = 1.531$.

Tetraquark	$J^{pc} = 2^{++}$	$J^{pc} = 1^{++}$	$J^{pc} = 0^{++}$
Masses:	$10341\,MeV$	$10303\,MeV$	$9936\,MeV$
$(b\bar{b})_{1^{--}}(u\bar{u})_{1^{--}}$	54.54	44.64	17.00
$(u\bar{b})_{1^{--}}(b\bar{u})_{1^{--}}$	1.36	1.57	0.44
$(b\bar{b})_{0^{-+}}(u\bar{u})_{0^{-+}}$	0	0	66.12
$(u\bar{b})_{0^{-+}}(b\bar{u})_{0^{-+}}$	0	0	0.72
Masses:	$10552\,MeV$	$10370\,MeV$	$10094\,MeV$
$(b\bar{b})_{1^{--}}(s\bar{s})_{1^{--}}$	48.65	37.14	16.86
$(s\bar{b})_{1^{--}}(b\bar{s})_{1^{--}}$	2.42	2.62	0.68
$(b\bar{b})_{0^{-+}}(s\bar{s})_{0^{-+}}$	0	0	63.58
$(s\bar{b})_{0^{-+}}(b\bar{s})_{0^{-+}}$	0	0	1.12

Table II. The widths, overlap factors f and phase spaces ρ of tetraquarks with hidden bottom.

Tetraquark	(channels)	J^{pc}	f	ρ	widths (MeV)
$X_b(9940)$	$\eta\eta_b$	0^{++}	0.66	0.0629	96
$X_b(10300)$	$Y\rho$	1^{++}	0.45	0.0617	43
$X_b(10340)$	$Y\omega$	2^{++}	0.55	0.0775	80
$X_b(10550)$	$Y\varphi$	2^{++}	0.49	0.0700	58

Table III. Masses, widths, overlap factors f and phase spaces ρ of scalar tetraquarks with open bottom.

Parameters of model: quark masses $m_u = 385\,MeV$, $m_s = 510\,MeV$, $m_b = 4787\,MeV$, cutoff parameter $\Lambda = 7.63$, gluon coupling constant $g = 1.531$.

Tetraquark	(channels)	f	ρ	Mass (MeV)	Widths (MeV)
$X_b(5910)$	$B^+\eta$	0.54	0.103	5914	104
$X_b(6020)$	$B_s^0\eta$	0.32	0.108	6017	69
	B^+K^-	0.23	0.171		
$X_b(6120)$	$B_s^0K^+$	0.283	0.174	6122	48

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